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Computerized Algebraic Utilities for the Construction of Nonsingular Satellite Theories

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An investigation was performed into the possibilities offered to celestial mechanics by a general purpose symbolic and algebraic manipulation language, in contrast to the classically used Poisson series processors. A package of blocks (routines) has been implemented on the MACSYMA system in order to develop a nonsingular semianalytical theory for the motion of an artificial satellite. Basic blocks generate the special functions that appear in the expansion of the gravitational potential in terms of equinoctial elements. Application blocks generate the literal expressions for the short-periodics and analytically averaged equations of motion (resonant tesserals are included). User inputs include the degree and order of the harmonic and maximum power of the eccentricity. Use of the system will be illustrated.

Introduction

THE use of electronic computers for analytical developments in celestial mechanics is an issue which has been debated for about twenty years.

In March 1958, at the Celestial Mechanics Conference held at Columbia University and the Watson Scientific Computing Laboratory in New York,[1] one of the topics discussed concerned the construction of general purpose compilers capable of carrying out literal theories in celestial mechanics. Since then, two different categories of people have been involved in the construction of algebra systems: 1) engineers and physicists who wrote special purpose programs in order to solve their specific problems; and 2) computer scientists who tried to implement general purpose algebraic manipulation systems in order to solve a wide variety of problems.

Special purpose programs are written in a very rigid mathematical environment. In celestial mechanics, and more specifically for the construction of literal analytical theories of motion of satellites and planets, most of the programs written so far perform algebraic operations on Poisson series.[2] One of the very first special purpose programs in celestial mechanics was written by Herget and Musen[3] in 1958 in order to generate Cayley's tables.

General purpose algebraic manipulation languages, more complex than special purpose programs since they have a larger application field, have appeared later. The first high-level non-numeric compiler, FORMAC[4] (FORmula MAnipulation Compiler), was implemented in 1965 at IBM in Cambridge, Massachusetts for the IBM 700 and 7000 series computers. As an application of FORMAC, Sconzo[5] calculated the f and g series used in certain expansions of elliptic motion to f_{27} and g_{27}. The two main drawbacks of early FORMAC were the lack of space for programs in execution and the absence of a few basic simplifications.[6]

There has been a lot of controversy about simplification in algebraic manipulation systems, due to a conflict of interests between the user and the designer. The user tries to produce expressions which have an easily understandable physical meaning for him; he would like the form of the expressions to depend on the context in which he is working. The designer would like to execute the same simplification steps whatever the context is. Thus, several types of algebraic manipulation systems have appeared, corresponding to different politics of simplification, which Moses[7] classified into five different categories: the Radicals, the New Left, the Liberals, the Conservatives, and the Catholics. The Radicals represent the designer's position—all expressions are put into canonical form, which insures a great computational efficiency. The Conservatives represent the user's position—most of the simplifications are under the user's control, which leads to very slow systems. Between these extremes, the three other types represent compromises. In particular, Catholic systems provide the user with several simplifiers, so that he may select the most appropriate, according to the form of the expressions he wants to simplify. The drawbacks of a Catholic system are its size, complexity, and inefficiency. The numerous services provided by such a system tend to slow it down and to increase the user's learning difficulties.

In celestial mechanics, it is remarkable to observe that, in spite of the possibilities offered by general purpose algebraic languages such as CAMAL,[8] the most important works have been performed using special purpose programs or radical systems such as MAO,[9] ESP,[10] and TRIGMAN.[11] One reason for this is that Poisson series possess two properties which make them extremely suitable for rigid systems: 1) they are closed under the operations of addition, subtraction, multiplication, and differentiation; and 2) to each Poisson series corresponds a canonical form which fully characterizes it.

Many Poisson series processors have been developed to treat problems in celestial mechanics; to give an exhaustive list of them would be illusory, but the works performed by Hall and Cherniack (SPASM),[12] Danby et al.,[13] Barton,[14] Chapront et al.,[15] Kutuzov,[16] and Broucke[17] may be given as examples.

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Most of the programs developed in celestial mechanics so far have been mainly concerned with computational efficiency. This was accomplished at the expense of the flexibility of the programs; their rigidity led to a lack of communication since these programs were written for a given machine and in a specific context. The direct consequence of that situation was the proliferation of Poisson series manipulators which have appeared during the last twenty years, although most of them have several common features.

Seven years ago, Jefferys[11] raised the question: "How will programs of use to celestial mechanics compare with general algebraic packages such as FORMAC, REDUCE, MATHLAB, SYMBAL?" After a brief discussion, he concluded that the "general systems were unsuitable for a particular problem because of the cost of time and space" but added that the situation might well change in the years to come, due to the decrease in hardware costs. Presently, the computational efficiency has become less critical and one is allowed to consider seriously the use of general systems in place of special purpose programs. Not only could a much larger community benefit from such systems, but also researchers would be freed from the tedious and time-consuming task of building their own systems.

This paper investigates the possibilities offered to celestial mechanics by a general algebraic manipulation system, MACSYMA. MACSYMA, Project MAC's SYmbolic MAnipulator,[18,19] is a large symbolic and algebraic manipulation system that has been under development at the Laboratory for Computer Science of the Massachusetts Institute of Technology since 1969. It is one of the most sophisticated Catholic systems, and very important efforts were made in its implementation to minimize the efficiency losses in running time and working storage space resulting from its size. MACSYMA is an interactive language which allows for a very good interface between the scientific user and the machine. Since May 1972, MACSYMA has been nationally available over the ARPA network[20] and, therefore, can be used by a large community.

To demonstrate the capabilities of the system, a package of blocks (routines) has been implemented on MACSYMA system in order to carry out the analytical part of a first-order semianalytical theory based on the expansion of the gravitational potential in terms of equinoctial variables.[21] The so-called "fundamental" blocks generate the special functions appearing in this expansion, while the "application" blocks produce the potentials, averaged equations of motion, and short-periodics due to an arbitrary harmonic, to the first order.

As an illustration of the possibilities offered by the MACSYMA satellite theory package, the critical inclination problem is chosen as an example. The theoretically predicted variations of eccentricity for Cosmos 373 are compared with actual observation data. The dramatic importance of the high-degree, odd-zonal harmonics in the vicinity of the critical inclination is graphically illustrated.

Semianalytical Theory

A semianalytical theory is chosen to treat the artificial satellite problem—rather than a complete analytical theory or a numerical theory—to test the possibilities provided by MACSYMA. Complete analytical theories constructed with presently available methods lead to the derivation of literal formulas for the orbital elements, which provide the investigator with a good insight of the physical phenomena. However, they require a huge storage capacity and are necessarily based on very restrictive assumptions for complexity reasons. Numerical theories, on the other hand, are very flexible, but they require starting the computations all over again for each new set of input data and they lose all the insight provided by analytical theories. A semianalytical theory, combining the features of an analytical theory and a numerical theory, is a desirable compromise.

A variation-of-parameters (VOP) formulation of the orbit prediction problem under the action of nonspherical gravitational perturbations is considered. As the perturbing accelerations are much smaller than the central force term, a VOP formulation is most suitable. In addition, the decoupling characteristics of the VOP equations are very useful in solving these differential equations.

The method of averaging is used in preference to a numerical integration directly applied to the osculating equations of motion. Indeed, in typical output scenarios, the method of averaging is believed to be significantly less costly than a high-precision technique.[22]

To avoid the appearance of singularities in the VOP equations of motion for vanishing eccentricity and/or inclination, an expansion of the gravitational potential in terms of the equinoctial elements is utilized.[21] The equinoctial elements are defined in terms of the classical orbital elements as follows:[21]

$$\left.\begin{array}{ll} a=a & p=\tan(i/2)\sin\Omega \\ h=e\sin(\omega+\Omega) & q=\tan(i/2)\cos\Omega \\ k=e\cos(\omega+\Omega) & \lambda=M+\omega+\Omega \end{array}\right\} \quad (1)$$

Analytical expressions of the averaged equations of motion are produced to the first order in a small parameter, requiring a numerical integration in order to determine the long period and secular motion of the orbital elements.

Once the mean elements are determined, corrective terms must be added which correspond to the short-period components. First-order analytical formulas have been derived by Cefola and McClain[23] for these corrective terms, expressed as functions of the mean elements, allowing a first-order recovery of the complete orbital elements as a function of time.

Terms due to $J_2{}^2$ are also included, although they are developed in a less automatic way.

MACSYMA Satellite Theory Package

A computerized algebraic utility has been implemented on MACSYMA in the form of ready-made protocols consisting of blocks in the sense of MACSYMA. Figure 1 shows the organization of the utility and makes the distinction between two types of blocks—the fundamental blocks and the application blocks.

The fundamental blocks generate the special functions appearing in the expansion of the gravitational potential in terms of equinoctial variables.[21] The scope of these blocks is not limited to the semianalytical theory considered in this paper. Indeed, certain of these special functions, such as the Newcomb operators or the Hansen coefficients, are extensively used in celestial mechanics. Moreover, such well-known functions as the inclination and eccentricity functions which appear in the expansion of the gravitational potential in terms of the classical orbital elements,[24] can easily be derived from the fundamental blocks. The inclination functions can be expressed in terms of the special functions defined in Ref. 21 in the following way:[25]

$$F_{nmp}(i) = V^m_{n,(n-2p)}\, S^{(m,n-2p)}_{2n}\,[0,\tan(i/2)] \qquad (n-m)\,\text{even} \quad (2a)$$

$$F_{nmp}(i) = jV^m_{n,(n-2p)}\, S^{(m,n-2p)}_{2n}\,[0,\tan(i/2)] \qquad (n-m)\,\text{odd} \quad (2b)$$

The eccentricity functions can be expressed in terms of the modified Hansen coefficients[21] according to the well-known relation[26]

$$G_{lpq}(e) = Y^{-l-1,l-2p}_{l-2p+q}(0,e) \qquad (3)$$

The application blocks, which are based on previous blocks, can generate the averaged zonal and resonant

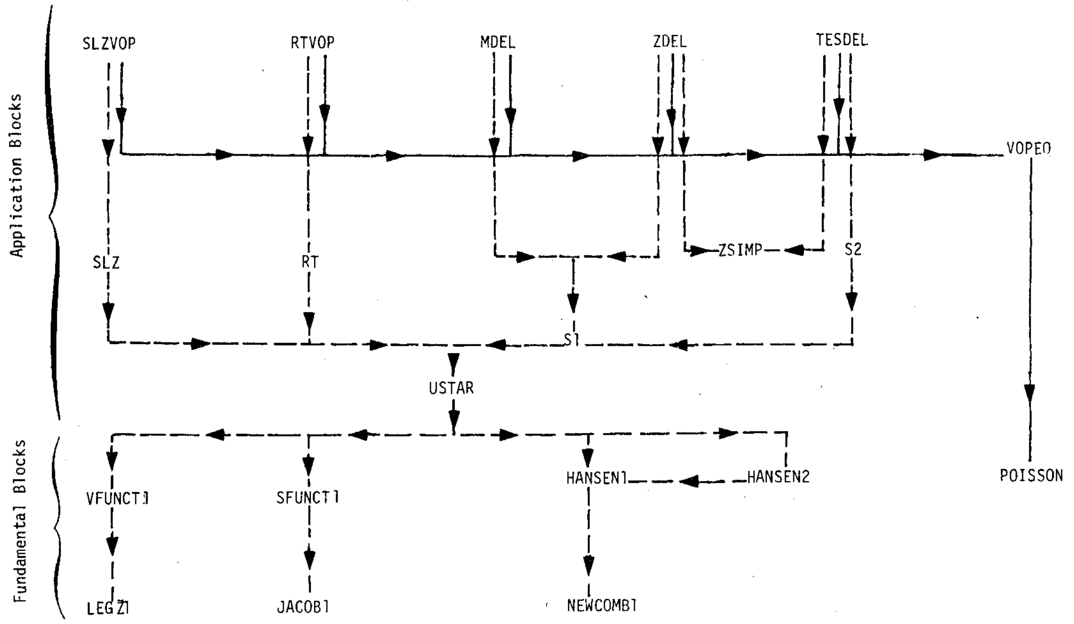

Fig. 1 Flow chart of the MACSYMA satellite theory package.

potentials, the averaged equations of motion, and the short-periodics due to any harmonic to the first order in a small parameter.

The fundamental blocks are stored on a file characterized by the two names, EGZ, POT2; the application blocks are stored on another file defined by the names EGZ, APL4. Both files are written on a disk under the directory called ZEIS. The names and functions of the different blocks are given in Table 1. Complete listings of the MACSYMA code with examples are given in Ref. 25.

For consistency, since the oblateness of the Earth J_2 is much larger than any other harmonic, the effects due to J_2 must be evaluated to the second order in that parameter. Because of time considerations, the derivation of the $J_2{}^2$ terms was not made in an automated way, but was performed according to an analyst-in-the-loop approach, with a truncation at the zeroth order in terms of the eccentricity. The second-order averaged equations of motion and short-periodics due to J_2 were determined by using the generalized method of averaging.[27]

The Critical Inclination Problem

To illustrate the capabilities offered by the MACSYMA satellite theory package, the critical inclination problem is chosen as an example. First, an extension of the classical definition of the critical inclination is made, based on the expressions of the averaged equations of motion provided by the satellite theory package. Second, an application to a real world problem is performed: the analysis of a near-critical inclination orbit, that of Cosmos 373. Finally, the importance of the high-order, odd-zonal harmonics is graphically illustrated.

Only the effects due to the long-period terms of the axially symmetric part of the geopotential are taken into consideration. Small eccentricity orbits are assumed and, therefore, the averaged equations of motion are dramatically truncated in terms of the eccentricity. The first-order contribution due to the oblateness of the Earth J_2 is expanded through the first order in terms of the eccentricity, while the second-order contribution due to J_2 and the first-order contributions due to the other zonal harmonics are truncated at the zeroth order in terms of the eccentricity. A geopotential model including the zonal harmonics up to J_9 is utilized. The first-order averaged equations of motion due to an arbitrary zonal harmonic are produced by using the block SLZVOP; the second-order averaged equations of motion due to J_2 are retrieved from a file where they were stored after having been manually derived. A careful examination of the different sets of averaged equations of motion shows that the global slowly varying equinoctial element rates can be expressed under the form

$$\dot{a}=0 \tag{4a}$$

$$\dot{h}=A'k+B'p \tag{4b}$$

$$\dot{k}=-A'h+B'q \tag{4c}$$

$$\dot{p}=C'q \tag{4d}$$

$$\dot{q}=-C'p \tag{4e}$$

where the coefficients A', B', and C' depend only on the semimajor axis a and the auxiliary variable $c=p^2+q^2$. Under the single influence of the axially symmetric part of the geopotential, the mean semimajor axis a is constant, according to relation (4a). The auxiliary variable c is also constant since, according to relations (4d) and (4e)

$$\dot{c}=2(p\dot{p}+q\dot{q})=2(C'pq-C'pq)=0 \tag{5}$$

Relations (4d) and (4e) show that p and q constitute a harmonic oscillator with an angular frequency C':

$$p=\alpha_1\cos C't+\alpha_2\sin C't \tag{6a}$$

$$q=\alpha_2\cos C't-\alpha_1\sin C't \tag{6b}$$

Relations (4b) and (4c) show that h and k constitute a harmonic oscillator with an angular frequency A', forced by oscillations with an angular frequency C'.

If $C'\neq\pm A'$, the general expressions for the equinoctial variables h and k are

$$h=\beta_1\cos A't+\beta_2\sin A't-\frac{B'}{C'-A'}(\alpha_2\cos C't-\alpha_1\sin C't) \tag{7a}$$

Table 1 Blocks of the MACSYMA satellite theory package

Name	Function
LEGZ1[N,R]	Computes the value of the associated Legendre function of degree N and order R at the origin
VFUNCT1[N,R,M]	Computes the rational number[21] $V^M_{N,R}$
JACOB1[N,A,B](Y)	Computes the Jacobi polynomial $P^{A,B}_N(Y)$
SFUNCT1[N,M,S](P,Q)	Computes the function[21] $S^{M,S}_{2N}(P,Q)$
NEWCOMB1[R,S,N,M]	Computes the Newcomb operator[28] $X^{N,M}_{R,S}$
HANSEN1[T,N,M,MAXE](H,K)	Computes the modified Hansen coefficient[21] $Y^{N,M}_T(H,K)$ with an expansion through order MAXE in terms of the eccentricity
HANSEN2[N,M,MAXE](H,K)	Computes the modified Hansen coefficient[21] $Y^{N,M}_0(H,K)$ with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
POISSON(I,J,MAXE)	Computes the Poisson bracket[27] (I,J) of the equinoctial elements I and J with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
USTAR[N,M,I,T,MAXE](A,H,K,P,Q,L)	Computes the complex potential[21]: $$U^*_{NMIT} = C^*_{NM}\ V^M_{N,I}\ S^{(M,I)}_{2N}(P,Q)\ Y^{-N-1,I}_T(H,K)\ \exp[j(TL-M\Theta)]$$ with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if T = 0 and MAXE < 0
SLZ[N,MAXE](A,H,K,P,Q,L)	Computes the averaged zonal potential due to the zonal harmonic of degree N with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
RT[N,M,RATIO,MAXE](A,H,K,P,Q,L)	Computes the averaged resonant potential due to the tesseral harmonic of degree N and order M for an orbit whose period is RATIO days with an expansion through order MAXE in terms of the eccentricity
VOPEQ(MAXE)	Computes the orbital element rates due to a disturbing potential R(A,H,K,P,Q,L) with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
SLZVOP[N,MAXE]	Computes the element rates due to the averaged zonal potential of degree N with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
RTVOP[N,M,RATIO,MAXE]	Computes the element rates due to the resonant potential of degree N and order M for an orbit whose period is RATIO days with an expansion through order MAXE in terms of the eccentricity.
S1[N,M,I,T,MAXE]	Computes the generator[23] $$S_{NMIT} = \frac{\int^{TL-M\Theta} \mathrm{Real}\{U^*_{NMIT}\}\, d(TL-M\Theta)}{Tn - M\,\omega_e}$$ with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if T = 0 and MAXE < 0
S2[N,M,I,T,RATIO,MAXE]	Identical to S1[N,M,I,T,MAXE] except that it conventionally assigns zero to the terms corresponding to the M-daily effect and the resonance effect for an orbit whose period is RATIO days
ZSIMP(EXPR)	Writes EXPR under the form of a Poisson series in the trigonometric variables L and Θ
MDEL[N,M,MAXE]	Computes the short-periodics due to the M-daily potential of degree N with an expansion through order MAXE in terms of the eccentricity if MAXE ≥ 0 or exactly if MAXE < 0
ZDEL[N, MAXE]	Computes the short-periodics due to the zonal harmonic of degree N with an expansion through order MAXE in terms of the eccentricity
TESDEL[N,M,RATIO,MAXE]	Computes the short-periodics due to the tesseral harmonic of degree N and order M for an orbit whose period is RATIO days with an expansion through order MAXE in terms of the eccentricity; the M-daily terms are not included.

NOTE: The nomenclature used in Table 1 is the following:

(A,H,K,P,Q,L) ≡ equinoctial elements (a,h,k,p,q,λ)

Θ ≡ Greenwich hour angle

n ≡ Mean motion

ω_e ≡ Earth's rotation rate

$$k=\beta_2\cos A't-\beta_1\sin A't+\frac{B'}{C'-A'}(\alpha_1\cos C't-\alpha_2\sin C't) \tag{7b}$$

If $C'=-A'$, h and k are under the form

$$h=\beta_1\cos A't-\beta_2\sin A't \tag{8a}$$

$$k=-(\beta_2+\frac{B'}{A'}\alpha_1)\cos A't-(\beta_1-\frac{B'}{A'}\alpha_2)\sin A't \tag{8b}$$

If $C'=A'$, the general solution for the equinoctial variables h and k is of the form

$$h=\beta_1\cos A't+\beta_2\sin A't+B'tp \tag{9a}$$

$$k=\beta_2\cos A't-\beta_1\sin A't+B'tq \tag{9b}$$

When C' approaches A', the amplitude of the oscillations of eccentricity with time becomes extremely large, according to Eqs. (7), but, at the same time, the period of the oscillations is also becoming very long. Thus the evolution of eccentricity with time tends to the unbounded secular motion described by Eqs. (9). This phenomenon is strictly similar to the mechanical resonance which is observed when a structure is excited by a frequency approaching one of its natural frequencies. The coefficients A' and C' depend only on the parameters a and $c=\tan^2 i/2$; therefore, corresponding to a semimajor axis a, critical inclinations i_c can be defined as the inclinations for which the resonance condition is realized:

$$A'(a,i_c)=C'(a,i_c) \tag{10}$$

The classical definition of the critical inclination can be recovered when only the first-order perturbations due to the oblateness of the Earth J_2 are taken into consideration. In this case, the coefficients A' and C' are

$$A'=\frac{3J_2(3c^2-6c+1)\mu R_e{}^2}{2a^5(c+1)^2 n} \tag{11a}$$

$$C'=\frac{3J_2(c-1)\mu R_e{}^2}{2a^5(c+1)n} \tag{11b}$$

where μ is the Earth's gravitational constant, R_e is the Earth's mean equatorial radius, and n is the mean motion of the satellite.

Equating relations (11a) and (11b) yields

$$c^2-3c+1=0 \tag{12}$$

Substituting for c its expression in terms of the inclination

$$\tan^2\frac{i}{2}=\frac{1-\cos i}{1+\cos i}$$

in Eq. (12) gives

$$5\cos^2 i-1=0 \tag{13}$$

which is the relation defining the classical critical inclination.

An analytical tool has been built using the satellite theory utility and other MACSYMA functions to predict the eccentricity variations of small eccentricity, near-critical inclination orbits (see Ref. 25 for details). Cosmos 373 is chosen as a real-world application because its inclination (62.91 deg) is close to the critical inclination (63.43 deg), and its eccentricity ($\sim 10^{-3}$) is small.

Brookes,[28] starting from actual observation data, recovered the eccentricity oscillations of Cosmos 373 caused by the axially symmetric part of the geopotential. To achieve this result, he first removed the luni-solar perturbations and then corrected for the effects of air drag by assuming a spiral vibration of eccentricity with the argument of perigee in the presence of air drag. The theoretical variations of eccentricity, $e=\sqrt{h^2+k^2}$, are obtained by assuming for the equinoctial variables h and k the form given by Eq. (7), where the constants are determined by the initial conditions. The semimajor axis a and the inclination i are considered as constant and equal to the mean values $a=6866$ km and $i=62.91$ deg. The corrected data points are in agreement with the theoretically predicted values of the eccentricity within 2% (Fig. 2), which is a reasonable accuracy given the approximations made. The theoretical prediction includes the zonal harmonics up to J_9.

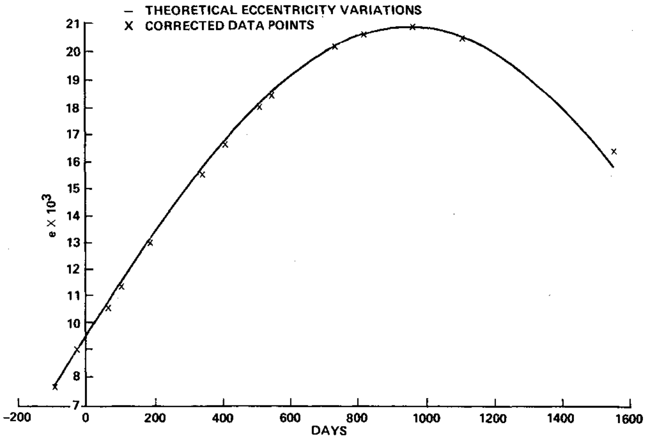

Fig. 2 Long-term and secular eccentricity variations for Cosmos 373 (a = G866 km, i = 62.91 deg).

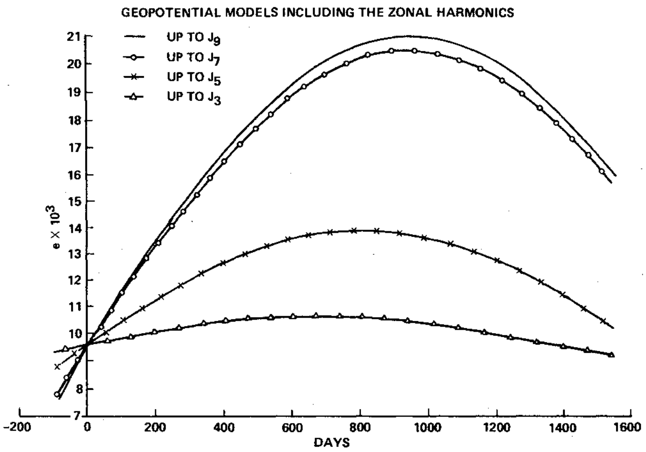

Fig. 3 Influence of the odd-zonal harmonics on the eccentricity variations for Cosmos 373 (a = 6866 km, i = 62.91 deg).

In order to determine the influences of the different odd-zonal harmonics, the theoretical eccentricity variations corresponding to geopotential models including zonal harmonics up to J_3, J_5, J_7, and J_9 are evaluated and plotted against time in Fig. 3. The dramatic importance of the high-order, odd-zonal harmonics J_5 and J_7, in the vicinity of the critical inclination, clearly appears in the figure. In order to recover most of the eccentricity variations, it is necessary to take into account the odd-zonal harmonics at least up to J_7, inclusive. On the other hand, the influence of J_3 seems surprisingly weak, which can be explained by considering the J_3 contributions to the coefficient B':

$$B'[J_3]=-\frac{3J_3(c^2-3c+1)\mu R_e{}^3}{a^6(c+1)^3 n} \tag{14}$$

The polynomial (c^2-3c+1) which appears in the numerator is precisely the left-hand side of Eq. (12) which defines the classical critical inclination. Therefore, in the vicinity of the critical inclination, the polynomial $(c^2 - 3c + 1)$ takes very small values and the J_3 contribution decreases accordingly. It happens that for inclinations close to the critical inclination, and greater than it, the J_3 contribution to B' is opposite in sign to the J_5, J_7, and J_9 contributions. As the inclination departs from the critical inclination, the relative importance of J_3 keeps increasing until it compensates exactly the influence of the other odd-zonal harmonics. Thus there exists an inclination close to the critical inclination such that, under the single influence of the axially symmetric part of the geopotential, the eccentricity remains frozen to its original value. For a semimajor axis $a = 6866$ km, as in the case of Cosmos 373, the inclination insuring a zero eccentricity rate was found equal to $i = 67.37$ deg.

Conclusions

In this paper, an investigation has been made concerning the possibilities offered to celestial mechanics by a general purpose algebraic language, MACSYMA. The utilization of the MACSYMA system itself and also the possible further developments of the MACSYMA satellite theory package are discussed.

The difficulties of using MACSYMA were classified by Genesereth[29] into three different categories: the learning difficulties, the resource knowledge difficulties, and the communication difficulties. The MACSYMA primer[30] and the reference manual[18] are very useful in resolving these difficulties. However, a manual describing the numerous interactions between the different MACSYMA functions and switches would be very desirable for the user constructing a large application system. The mathematical knowledge and sophisticated commands built into the system make it possible to write very compact and easily readable blocks at the expense of small programming work.

Several directions of research are foreseen for the future work. In the short term, some effort must be spent to improve the efficiency of the MACSYMA satellite theory package. The presentation of the outputs is certainly a key issue, because a good deal of time is spent in simplifying the expressions produced and making them as compact as possible. A careful examination of the simplification scheme would probably lead to significant improvements in the efficiency. The use of the TRANSLATE command to create a LISP version of the blocks would result in a gain in speed. Finally, the use of the CLOCK command seems very promising; this command indicates the time spent in all the functions declared as arguments to CLOCK. When calling a high-level block, the command CLOCK will determine the respective times spent in the different lower-level blocks called in this high-level block. Thus, the command CLOCK will allow a better understanding of the mechanism of the computation and eventually lead to an improved structure of the whole package.

Additional work must be performed in the testing of the MACSYMA satellite theory package. While the fundamental blocks and the application blocks generating the disturbing potentials and averaged equations of motion have been checked extensively, the blocks generating the tesseral short-periodics (TESDEL) need more testing.

In the short term, the future work will also include the derivation of the second-order disturbing effects due to the oblateness of the Earth J_2 in a more efficient and automated way. To achieve this result, a Lagrangian formulation of the VOP equations of motion is much more appropriate than the general Gaussian formulation that has been utilized. A formulation that is closed in the eccentricity should be developed.

In the long term, future work should include a comparative study of different nonsingular orbital elements such as the ideal elements.[31] Some orbital element sets seem to lead to more compact final expressions and cause less intermediate expression swell than the equinoctial element set. The work of Deprit[32] is noted.

Finally, future work should also include an investigation of the mission design applications of the MACSYMA satellite theory package, comparable to the critical inclination problem.

In conclusion, the MACSYMA satellite theory package, whose capabilities have been illustrated by the critical inclination problem, reveals itself as a quite encouraging experiment for the use of general purpose algebraic languages in celestial mechanics. Before pursuing it further, however, and possibly developing a second-order semianalytical artificial satellite theory, it is essential to take advantage of the knowledge of MACSYMA acquired so far and spend some time optimizing the analytical tool which has been developed.

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